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Ion Trap Quantum Computing with Ca^+ Ions

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The scheme of an ion trap quantum computer is described and the implementation of quantum gate operations with trapped Ca^+ ions is discussed. Quantum information processing with Ca^+ ions is exemplified with several recent experiments investigating entanglement of ions.

KEY WORDS: Quantum computation; trapped ions; ion trap quantum computer; Bell states; entanglement; controlled-NOT gate; state tomography; laser cooling.

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1. INTRODUCTION

Quantum information processing was proposed and considered first by Feynman and Deutsch.(1,2) The requirements for a quantum processor are nowadays known as the DiVincenzo criteria.(3) Storing and processing quantum information requires: (i) scalable physical systems with well-defined qubits; which (ii) can be initialized; and have (iii) long lived quantum states in order to ensure long coherence times during the computational process. The necessity to coherently manipulate the stored quantum information requires: (iv) a set of universal gate operations between the qubits which must be implemented using controllable interactions of the quantum systems; and finally, to determine reliably the outcome of a quantum computation (v) an efficient measurement procedure. In recent years, a large variety of physical systems have been proposed and investigated for their use in

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quantum information processing and are considered in other articles of this issue.

In this paper, quantum information processing is discussed using trapped Ca^+ ions where the qubit is encoded in long-lived (ground and metastable) electronic states. A possible different approach for encoding qubits uses two hyperfine levels of hydrogen-like ions, e.g., Be^+ ions (see article by Blinov *et al.*[4] in this volume) or Cd^+ ions.[5]

This paper is organized as follows: after a brief introduction to the concept of the ion trap quantum computer in Sec. 2, some crucial details of the Ca^+-based approach are outlined in Sec. 3. Coherent manipulation of the ions is briefly described in Sec. 4 and the basic two-ion gate operation is reviewed in Sec. 5. The preparation of two-qubit entangled states is summarized in Sec. 6 and future developments of a Ca^+-based ion trap computer are outlined in Sec. 7.

2. CONCEPT OF THE ION TRAP QUANTUM COMPUTER

Strings of trapped ions were proposed in 1995 for quantum computation by Ignacio Cirac and Peter Zoller.[6] With such a system, all requirements for a quantum information processor[3] can be met. Using strings of trapped ions in a linear Paul trap, qubits can be realized employing either metastable excited states, long-lived hyperfine states or corresponding Zeeman sub-states. A set of universal quantum gate operations is then given by: (i) single-qubit rotations (which are realized by Rabi oscillations of individual ions); (ii) the controlled-NOT (CNOT) operation between any two qubits. As a first step the entire ion string is cooled to the ground state of its harmonic motion in the ion trap. Since the mutual Coulomb repulsion spatially separates the ions, any induced motion couples to all ions equally. By applying a laser pulse to the controlling ion its internal excited state amplitude is mapped to a single phonon quantum motion of that ion. This phonon, however, is now carried by the entire string, and an operation on the target qubit which depends on whether or not there is motion in the string, allows one to realize the CNOT-gate operation.

Any algorithm can be implemented using a series of such one- and two-qubit operations and therefore this set of instructions constitutes a universal quantum gate.[7] Thus, the realization of these quantum gates allows one to build and operate a quantum computer. Moreover, in principle, this concept provides a scalable approach towards quantum computation and has therefore attracted quite some attention.

During recent years, several other techniques have been proposed to implement gate operations with trapped ions. Sørensen and Mølmer

[8,9] and, with a different formulation, Milburn[10] proposed a scheme for "hot" quantum gates, i.e., their procedures for gate operations do not require ground state cooling of an ion string. Although successfully applied to trapped Be$^+$ ions,[11] with the trapping parameters currently available, these gate procedures are not easily applicable to Ca$^+$ ions. Other gates based on ac Stark shifts have been suggested by Jonathan *et al.*[12] and holonomic quantum gates (using geometric phases) have been proposed by Duan *et al.*[13] A different CNOT-gate operation also based on the ac Stark effect which does not require individual addressing and ground state cooling has been realized with trapped Be$^+$ ions.[14]

3. SPECTROSCOPY IN ION TRAPS

Ions are considered to be trapped in a harmonic potential with frequency ν_z, interacting with the travelling wave of a single mode laser tuned close to a transition that forms an effective two-level system.

Internal state detection of a trapped ion is achieved using the electron shelving technique. For this, one of the internal states of the trapped atom is selectively excited to a third short-lived state thereby scattering many photons on that transition if the coupled internal state was occupied. If, on the other hand, the atom's electron resides in the uncoupled state of the qubit (i.e., the electron is shelved in that state) then no photons are scattered and thus the internal state can be detected with an efficiency of nearly 100%.[15]

Figure 1 shows the relevant levels of the Ca$^+$ ion which are populated in the experiment. The qubit is implemented using the narrow quadrupole transition at 729 nm, i.e., $|g\rangle = |S_{1/2}\rangle$ and $|e\rangle = |D_{5/2}\rangle$. For optical cooling and state detection, resonance fluorescence on the $S_{1/2}$–$P_{1/2}$ transition is

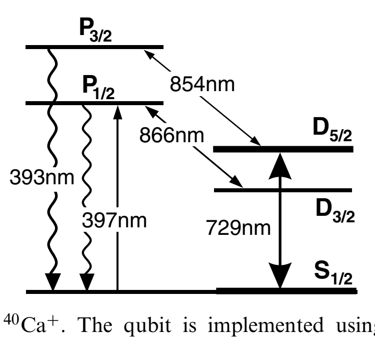


Fig. 1. Level scheme of $^{40}Ca^+$. The qubit is implemented using the narrow quadrupole transition. All states split up into the respective Zeeman sublevels.

scattered by excitation with 397 and 866 nm radiation. The laser at 854 nm is applied to repump the excited state $|e\rangle$, for example after a shelving operation.

3.1. Laser Cooling of Ion Strings

A prerequisite of the Cirac–Zoller (CZ) scheme is that the initial state of the quantum register is prepared in its motional ground state, i.e., we require that the motional mode which carries the coupling between the qubits is initially in the ground state.

Laser cooling of trapped ions is therefore one of the key techniques for an ion trap quantum computer.(15,16) Usually so-called sideband cooling(17,18) is used to cool one mode of an ion string to its motional ground state. This is experimentally achieved using optical pumping schemes involving either Raman transitions(19) or coupled transitions.(18,20) More elaborate cooling schemes using electromagnetic transparency(21,22) or sympathetic cooling(23–25) have been investigated and and can be employed for cooling multiple vibrational modes simultaneously or cooling an ion string by addressing just one ion, respectively.

3.2. Addressing of Individual Ions

The implementation of the CZ CNOT-gate operation requires that individual ions can be addressed in order to rewrite internal information onto the vibrational ("phonon") mode using appropriate transitions. Therefore, the Innsbruck experiments were designed to operate in a regime where the minimum ion distance is on the order of a few μm such that focussing a laser beam at 729 nm is feasible to individually address the single ions.(26) In the current setup, Ca^+ ions are stored with axial trap frequencies of about 1–1.2 MHz and thus the inter-ion distance of two and three ions is approximately 5 μm. The laser beam at 729 nm is focussed to a waist diameter of approximately 2.5 μm such that with the Gaussian beam profile neighboring ions are excited with less than 10^{-3} of the central intensity. Beam steering and individual addressing is achieved using electrooptic beam deflection which allows for fast switching ($\sim$15 μs) between different ion positions.(27)

4. COHERENT MANIPULATION OF QUANTUM INFORMATION

Quantum information processing requires that individual qubits are coherently manipulated. We realize single-qubit rotations by coherent manipulation of the $S_{1/2}(m=-1/2) \leftrightarrow D_{5/2}(m=-1/2)$ transition in Ca^+. Coupling of two qubits requires the precise control of the motional state of a single ion or a string of ions. Both operations can be performed by applying laser pulses at the carrier (i.e., not changing the vibrational quantum number, $\Delta n_z = 0$) or at one of the sidebands of the S–D transition (i.e., laser detuned by $\pm\nu_z$, thus changing the vibrational quantum number by $\Delta n_z = \pm 1$).

All qubit transitions are described as rotations on a corresponding Bloch sphere and they are written as unitary operations $R(\theta,\phi)$, $R^-(\theta,\phi)$, $R^+(\theta,\phi)$ on the carrier, red sideband and blue sideband, respectively. The parameter θ describes the angle of the rotation and depends on the strength and the duration of the applied pulse. ϕ denotes its phase, i.e., the relative phase between the optical field and the atomic polarization and determines the axis about which the Bloch vector rotates.(27) Typical pulse durations for a π-pulse range from about 1 to several 10 μs for the carrier transition and 50–200 μs on the sideband transition, with the chosen time depending on the desired speed and precision of the operations. Such pulses are the primitives for quantum information processing with trapped ions. By concatenating pulses on the carrier and sidebands, gate operations and, eventually whole quantum algorithms, can be implemented.(27) Even the simplest gate operations require several pulses, therefore it is imperative to control the relative optical phases of these pulses in a very precise manner or, at least, to keep track of them such that the required pulse sequences lead to the desired operations. This requires the precise consideration of all phases introduced by the light shifts of the exciting laser beams.(28)

5. CIRAC–ZOLLER CNOT-GATE OPERATION

For the realization of the CZ CNOT-gate operation, two ions are loaded into the linear trap and, by means of an intensified CCD camera, the fluorescence is monitored separately for each ion.(27) If no information on a particular qubit is needed, the signal of a more sensitive photomultiplier tube is used to infer the overall state population and, thus, the exposure time can be reduced.

As proposed by Cirac and Zoller, the common vibration of an ion string is used to convey the information for a conditional operation (bus-mode).(6) Accordingly, the gate operation can be achieved with a sequence of three steps after the ion string has been prepared in the ground state $|n_b = 0\rangle$ of the bus-mode. First, the quantum information of the control ion is mapped onto this vibrational mode. As a result, the entire string of ions is moving and thus the target ion participates in the common motion. Second, and conditional upon the motional state, the target ion's qubit is inverted. Finally, the state of the bus-mode is mapped back onto the control ion. Note that this gate operation is not restricted to a two-ion crystal since the vibrational bus-mode can be used to interconnect any of the ions in a large crystal, independent of their position.

We realize this gate operation[29] with the following sequence of laser pulses. A blue sideband π-pulse, $R^+(\pi, 0)$, on the control ion transfers its quantum state to the bus-mode. Next, we apply the CNOT-gate operation

$$R_{\mathrm{CNOT}} = R\left(\frac{\pi}{2}, 0\right) R^+\left(\pi, \frac{\pi}{2}\right) R^+\left(\frac{\pi}{\sqrt{2}}, 0\right) R^+\left(\pi, \frac{\pi}{2}\right) R^+\left(\frac{\pi}{\sqrt{2}}, 0\right) R\left(\frac{\pi}{2}, \pi\right) \quad (1)$$

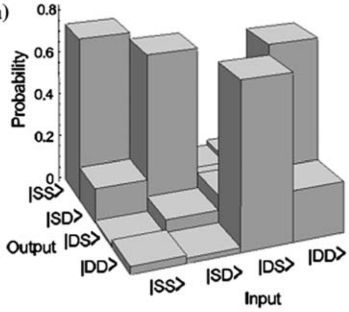
to the target ion. Finally, the bus-mode and the control ion are reset to their initial states by another π-pulse $R^+(\pi, \pi)$ on the blue sideband. The resulting gate fidelity of about 71–78% is well understood in terms of a collection of experimental imperfections.[29] Most important is dephasing due to laser frequency noise and ambient magnetic field fluctuations that cause a Zeeman shift of the qubit levels.[27] As quantum computing might be understood as a multi-particle Ramsey interference experiment, a faster execution of the gate operation would help to overcome this type of dephasing errors. However, a different type of error increases with the gate speed: with higher Rabi frequencies, the off-resonant excitation of the nearby and strong carrier transition becomes increasingly important,[30] even if the corresponding phase shift is compensated. Additional, but minor, errors are due to the addressing imperfection, residual thermal excitation of the bus-mode and spectator modes as well as laser intensity fluctuations.

If the qubits are initialized in the superposition state $|\text{control, target}\rangle = |S + D, S\rangle$, the CNOT operation generates an entangled state $|S, S\rangle + |D, D\rangle$. Using local operations with varying phase then allows the preparation of arbitrary Bell states using the CNOT-gate operation[31] (see Fig. 2).

6. BELL STATE GENERATION AND ENTANGLEMENT STUDIES

Bell states are very important for an investigation of entanglement with the capability to produce them at the push of a button is one of the major advantages of an ion trap quantum computer. However, while conceptually simple and straightforward, Bell states need not be generated

(a)



(b)

	$\vert SS\rangle$	$\vert SD\rangle$	$\vert DS\rangle$	$\vert DD\rangle$
$\vert SS\rangle$	0.74(3)	0.13(3)	0.05(3)	0.08(3)
$\vert SD\rangle$	0.15(3)	0.71(5)	0.06(1)	0.08(2)
$\vert DS\rangle$	0.01(2)	0.08(3)	0.14(4)	0.77(3)
$\vert DD\rangle$	0.03(3)	0.02(1)	0.72(6)	0.22(4)

Fig. 2. Truth table of CZ CNOT-gate operation. The amplitude of the controlling ion (first entry of the state notation) controls the state of the target ion (second entry), i.e., when the controlling ion's amplitude is $|S\rangle$, the target ion's state remains the same, when it is $|D\rangle$, the target's ion state is flipped. (a) graphical representation; (b) numerical results as shown in (a).[29]

using CNOT-gate operations. With trapped ions, there are simpler and more efficient procedures to produce and investigate these states.

Using a string of two ions and the individual addressing capability in the Innsbruck experiment, we create all Bell states by applying laser pulses to ions 1 and 2 on the blue sideband and on the carrier transition. To produce the Bell state $\Psi_\pm = 1/\sqrt{2}(|S, D\rangle \pm |D, S\rangle)$ we use the pulse sequence $U_{\Psi_\pm} = R_2^+(\pi, \pm\pi/2) \cdot R_2(\pi, \pi/2) \cdot R_1^+(\pi/2, -\pi/2)$ applied to the $|S, S\rangle$ state. Here, the indices 1 (2) refer to pulses applied to ions 1 and 2, respectively. The first pulse $R_1^+(\pi/2, -\pi/2)$ entangles the motional and the internal degrees of freedom. The next two pulses $R_2^+(\pi, \pm\pi/2) \cdot R_2(\pi, \pi/2)$ map the motional degree of freedom onto the internal state of ion 2. Appending another π-pulse on the carrier transition, $R_2(\pi, 0)$, to the sequence $U_{\Psi_\pm}$ produces the state $\Phi_\pm$. This entire pulse sequence takes less than $200\,\mu$s and is much simpler than a full CZ CNOT-gate operation which takes about twice that time and is thus more sensitive to decoherence.

Investigation of the prepared state and a characterization of the achieved entanglement then is obtained by a quantum state analysis using a tomographic procedure. Quantum state tomography allows the estimation of an unknown quantum state that is available in many identical copies. It has been experimentally demonstrated for a variety of physical systems, among them the quantum state of a light mode,[32] the vibrational state of a single ion,[33] and the wave packets of atoms of an atomic beam.[34] Multi-particle states have been investigated in nuclear magnetic resonance experiments[35] as well as in experiments involving entangled photon pairs. Here, we apply this technique to entangled massive particles of a quantum register for an investigation of entanglement and studies of decoherence.

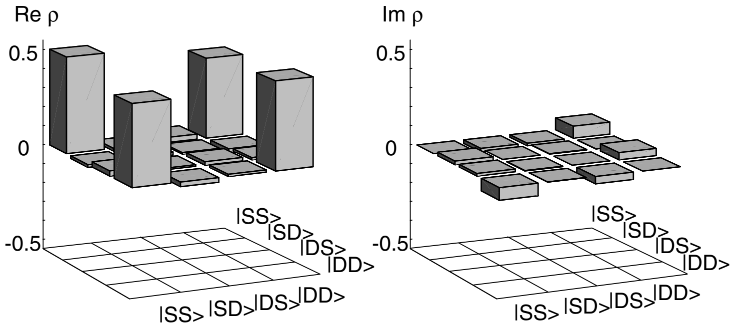


Fig. 3. Real and imaginary part of the density matrix ρ_{Φ_+} that approximates $1/\sqrt{2}(|S,S\rangle + |D,D\rangle)$. The measured fidelity is $F = \langle\Phi_+|\rho_{\Phi_+}|\Phi_+\rangle = 0.91$.

The tomographic method consists of individual single-qubit rotations, followed by a projective measurement. For the analysis of the data, we employ a maximum likelihood estimation of the density matrix following the procedure as suggested in Refs. 36 and 37 and implemented in experiments with pairs of entangled photons.(38) As an example, Fig. 3 shows the reconstructed density matrix ρ of one out of four Bell states. To monitor the evolution of these entangled states in time we introduce a waiting interval before performing state tomography. We expect that Bell states of the type $\Psi_\beta = |S,D\rangle + e^{i\beta}|D,S\rangle$ are immune against collective dephasing due to fluctuations of the qubit energy levels or the laser frequency.(39) However, a magnetic field gradient that gives rise to different Zeeman shifts on qubits 1 and 2 leads to a deterministic and linear time evolution of the relative phase $e^{i\beta}$ between the $|S,D\rangle$ and the $|D,S\rangle$ component of the $\Psi_\pm$ states. Experimentally, we find that the lifetime of entangled states of this type is indeed no longer limited by the technical constraints (i.e., magnetic field and laser frequency fluctuations) but is only limited by the spontaneous decay from the upper $D_{5/2}$-level (lifetime $\tau_D \simeq 1$ s) of the qubit. Finally, we can specify the entanglement of the four Bell states, using the entanglement of formation,(40) and find $E(\Psi_-) = 0.79(4)$, $E(\Psi_+) = 0.75(5)$, $E(\Phi_+) = 0.76(4)$ and $E(\Phi_-) = 0.72(5)$.(31)

7. FUTURE DEVELOPMENTS OF THE CA$^+$ ION TRAP QUANTUM COMPUTER

With the availability of one- and two-qubit operations, the individual addressing and the near perfect readout features, a Ca^+-based ion trap quantum computer can be envisioned. Currently, the techniques described above are extended to work with three and more ions which already offer a vast variety of experimental possibilities, ranging from the preparation and investigation of generalized 3-qubit entangled states to an implementation of teleportation and rudimentary error correction protocols.

While detection efficiencies and noise considerations are quite favorable for an optical qubit transition, there are a number of technical limitations. Most of these limitations are not of a fundamental nature, but are rather given by technical shortcomings, such as the sensitivity of the qubit transition with respect to external magnetic fields and spurious laser frequency and intensity fluctuations. The only fundamental limitation is the lifetime of the pertaining qubit states, here in particular that of the $D_{5/2}$ state (1.16 s), which, however, is orders of magnitude larger than typical gate operation times. The limitations discussed above might lead to reconsidering the use of ground state Zeeman and hyperfine splittings for encoding the quantum information. We illustrate here the specific pros and cons considering respective transitions in $^{40}Ca^+$ and $^{43}Ca^+$ ions. Whereas the current experiments work with an optical qubit (i.e. $|0\rangle = |D_{5/2}, m_J = -1/2\rangle$ and $|1\rangle = |S_{1/2}, m_J = -1/2\rangle$, cf. Fig. 1) in the even isotope $^{40}Ca^+$, an alternative implementation would work with the odd isotope $^{43}Ca^+$ (nuclear spin $I = 7/2$) and the hyperfine ground states $|0\rangle = |F = 4, m_F = 0\rangle$ and $|1\rangle = |F = 3, m_F = 0\rangle$ (see Fig. 4). In the latter case, optical manipulation of the qubit would be achieved using Raman transitions.

To a large extent the coherence properties of the qubits depend on the respective sensitivity on external field fluctuations, e.g., magnetic and laser field fluctuations. Therefore, in the optical case, a highly stabilized laser is required for the qubit transition whereas in the case of a Raman transition, both Raman beams can be derived from the same laser source where the required stable relative phase relation can be achieved with only modest technical efforts. The large fine-structure splitting of $\Delta\nu_{FS}$ = 6.7 THz between the $P_{1/2}$ and $P_{3/2}$ states allows a large detuning of the Raman light fields from the P-levels and thus high fidelity gate operations, as spontaneous emission processes are largely suppressed. The fine-structure splitting of $^{43}Ca^+$ can be compared to that of other favorable qubit candidates, e.g. $^9Be^+$ with $\Delta\nu_{FS}$ = 0.2 THz and $^{111}Cd^+$ with $\Delta\nu_{FS}$ = 74 THz.

Aside from these more technical constraints, encoding the qubit in the hyperfine ground states ensures that decay from spontaneous emission is completely avoided and thus, very long coherence times (many seconds and even minutes have been demonstrated with trapped Be^+ ions) may be potentially achieved. Furthermore, the qubits will, ideally, depend only in second order on the external magnetic field ($\Delta m_F = 0$ transitions, see Fig. 4). While many of these advantages are available already with Be^+ and Cd^+ ions, the $^{43}Ca^+$ ion offers additionally, a quadrupole

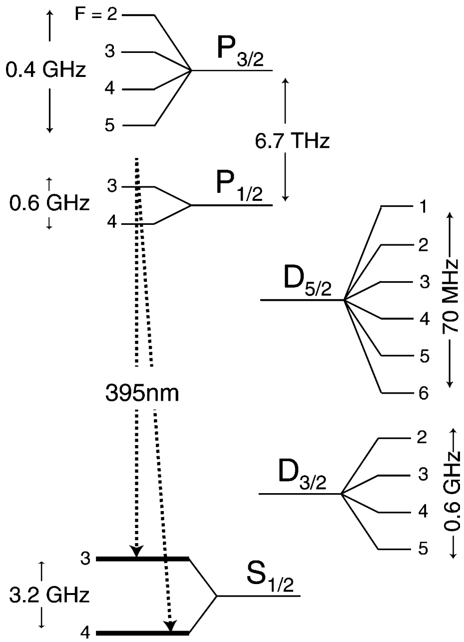


Fig. 4. Level scheme of the $^{43}Ca^{+}$ isotope. A qubit can be encoded in the hyperfine ground states $|0\rangle = |F = 4, m_F = 0\rangle$ and $|1\rangle = |F = 3, m_F = 0\rangle$.

transition that can be advantageously used for shelving and efficient detection without the need for a technically advanced laser source. Therefore, the next generation of a Ca^{+}-based ion trap quantum computer will ideally combine the advantages of the ground state encoding of the qubit and the optical shelving and detection techniques.

8. SUMMARY AND PERSPECTIVES

On the road towards a scalable quantum processor[(41)] with ion traps, single-qubit rotations and universal two-qubit operations gate have been realized. With trapped Ca^{+} ions, we present an experimental setup which allows one to flexibly control a register of two qubits. With the universal set of quantum gates all unitary operations can be implemented. Therefore, arbitrary two-qubit states can be synthesized with high fidelities and analyzed via state tomography. The currently available experiments demonstrate the operation of a small quantum computer and allow one to develop the basic tools of experimental quantum information processing.

One of the most striking features is that the ion trap quantum information processor is scalable in principle, i.e., adding more qubits is straightforward and at least up to about 10 qubits this should not pose insurmountable technical difficulties. Larger systems will require special architectures such as ion trap arrays,[42] moving ions in structured ion traps[43] or even interconnecting several small ion-trap quantum computers using cavities and photons as a quantum channel.[44,45] While all these techniques require tremendous technical efforts, to the best of our current knowledge there are no principal limitations to scaling up an ion-trap quantum computer.

The current experiments demonstrate that ion trap quantum information processors offer a realistic route towards the realization of large-scale quantum computing and they provide ideal means for the engineering of quantum objects and controlling quantum processes at mesoscopic and macroscopic scales.

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