Interfacing Quantum-Optical and Solid-State Qubits

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We present a generic model of coupling quantum-optical and solid-state qubits, and the corresponding transfer protocols. The example discussed is a trapped ion coupled to a charge qubit (e.g., Cooper pair box). To enhance the coupling and to achieve compatibility between the different experimental setups we introduce a superconducting cavity as the connecting element.

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Significant progress has been made during the last few years in implementing quantum computing proposals with various physical systems [1]. Prominent examples are quantum-optical systems, in particular, trapped ions and atoms in optical lattices [2,3], and more recently solid-state systems, such as Josephson junctions [4] and quantum dots [5]. While quantum optics *per se* provides stable qubits, high-fidelity quantum gate operations and read out, future advances in quantum optics will increasingly be based on incorporating ideas and methods from emerging nanotechnologies, i.e., the same technologies which underlie much of the present progress in solid-state quantum computing. An example is the development of mesoscopic segmented ion traps as a key to a scalable ion trap quantum computing [3], where lithographic traps allow the moving of individual ions (representing the qubits) between a storage and a processing unit. Going one step further, the question arises to what extent quantum optics and solid-state *hybrid systems* for quantum computing can be developed with the goal of combining advances of various approaches, while still being experimentally compatible (see also [6,7]). This provides the motivation to study the interfacing of quantum-optical qubits and solid-state systems: this can be in a form where quantum-optical qubits are connected via a solid-state data bus or by reversibly transferring quantum-optical to solid-state qubits and circuits.

Quantum-optical qubits are typically represented by internal (nuclear) spin states of single atoms or ions well isolated from environmental noise [8]; solid-state charge qubits can be operated at a nanosecond time scale via Coulomb interaction or exchange interaction [4,5]. In our scheme, the long-lived quantum-optical qubits are the quantum memory unit, and the fast operable solid-state charge states are the processing unit. Each call of logic gate includes three steps: (i) a swap gate is applied which transfers the state from the memory to the solid-state qubit; (ii) a gate operation is performed on the solid-state qubit(s); (iii) finally, a second swap gate is applied to transfer the processed state to the optical qubit.

In this Letter we discuss the generic physical model and the corresponding protocol for interfacing a quantum-optical and solid-state charge qubit. On the quantum-optical side, the single atoms can be trapped and laser cooled. External fields provide a mechanism to manipulate the qubit, as well as to entangle the qubit with the motional state of the trapped particle. In the case of trapped ions this allows one to convert spin (the qubit) to charge superpositions, either dynamically, e.g., by kicking with a laser, or quasistatically by applying spin-dependent (optical or magnetic) potentials. This spin to charge conversion provides a natural capacitive coupling to a solid-state charge qubit, represented, e.g., by a Josephson junction [4] or a charged double quantum dot [5]. Instead of direct coupling of the charges, one can introduce auxiliary elements, such as cavities. This serves the purpose of allowing increased spatial separation and mutual shielding of the systems, with the goal of easing experimental requirements for coexistence of the hybrid qubits (e.g., trapping and laser manipulation of atoms or ions), while enhancing the coupling strength. Features of this hybrid system are the significantly different time scales of the evolution and coupling of the two qubits, and (in comparison with quantum-optical qubits) a short decoherence time of the solid-state systems. We exploit this by developing a protocol of a "fast swap" gate on a time scale comparable to the charge-charge coupling and much shorter than the trap period. This has the additional benefit of being a hot gate, i.e., not requiring cooling to the motional ground state, and not making a Lamb-Dicke assumption of strong confinement.

Model for coupled qubits.—A Hamiltonian for our combined system has the form $H_t = H_s + H_q + H_{\rm int}$ with a Hamiltonian for the quantum-optical qubit

$$H_s = \left(\frac{\hat{p}_x^2}{2m} + \frac{1}{2} m\omega_\nu^2 \hat{x}^2\right) + \frac{\delta_0}{2}\sigma_z^s + \left(\frac{\hbar\omega_R(t)}{2}\sigma_+^s e^{i\delta k_l \hat{x}} + \text{H.c.}\right), \quad (1)$$

the solid-state charge qubit,

$$H_q = \frac{E_z}{2}\sigma_z^q + \frac{E_x}{2}\sigma_x^q, \tag{2}$$

and the interaction term

$$H_{\rm int} = \hbar\kappa(t)\hat{x}\sigma_z^q. \tag{3}$$

The first term in H_s describes the 1D motion of a charged particle (ion) in the harmonic trapping potential with $\hat{x}$ as the coordinate, $\hat{p}_x$ as the momentum, and ω_ν as the trapping frequency. A pseudospin notation with Pauli operators σ_i^s describes the atomic qubit. Physically, the qubit is represented by two atomic ground state levels which are coupled by a laser induced Raman transition with Rabi frequency $\omega_R(t)$ and detuning δ_0. Transitions between the states are associated with a momentum kick δk_l due to photon absorption and emission, which couples the qubit to the motion at the Rabi frequency $\omega_R(t)$ [9].

The Hamiltonian for the solid-state charge qubit H_q has the generic form for the quantum two level system with σ_i^q being the Pauli operators and $E_{x,z}$ tunable. A Hamiltonian of this form is obtained, for example, for a superconducting charge qubit, i.e., a superconducting island connected to a high resistance tunnel junction [see Fig. 1]. With the phase φ of the superconductor and its conjugate $\hat{n}$, the number of Cooper pairs on the island, the Hamiltonian is $H_q = E_c(\hat{n} + C_g V_g/2e)^2 - E_J\cos\varphi$ [4], where E_J is the Josephson energy and E_c is the capacitive energy with $E_c \gg E_J$. The gate voltage V_g controls the qubit through the gate capacitor C_g. When $V_g \sim (2m+1)e/C_g$ (m is an integer), the qubit forms an effective two level system with charge states $|0\rangle = |n\rangle$ and $|1\rangle = |n+1\rangle$, and in Eq. (2), $E_z = E_c(C_g V_g/2e)$ and $E_x = E_J$. Adjusting V_g or E_J provides arbitrary single-qubit gates. Typically, E_c is about 100 GHz and E_J is about 10 GHz. Other solid-state systems such as a double quantum dot qubit can be considered within a similar framework.

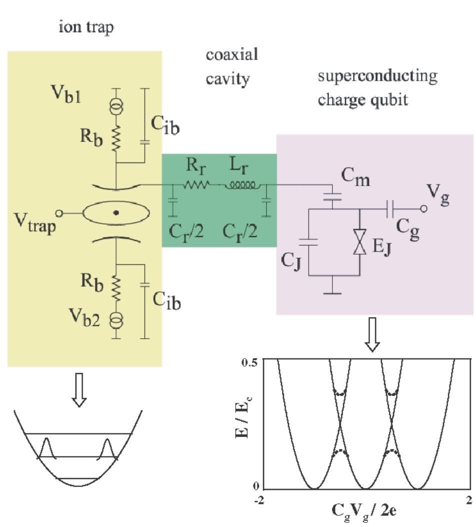

FIG. 1 (color online). Schematic coupling circuit of the charge qubit and trapped ion. Top: the coupling via a cavity. The voltage at the electrode is balanced by a filtering circuit. Bottom left: coherent states of the motional mode. Bottom right: energy of charge qubit vs gate voltage.

Finally, the interaction [Eq. (3)] has the universal form of the electrostatic coupling between a motional dipole and a charge which is linear in the coordinate $\hat{x}$ and the charge operator σ_z^q. Note this coupling is of the order of $ep_i/4\pi\epsilon_0 r_0^2$ for a given distance r_0 with dipole p_i and charge e and is a factor of er_0/p_i stronger than the familiar dipole-dipole couplings encountered in quantum optics. Instead of a direct coupling of the dipole to the charge, we introduce an interaction via short superconducting cavity. This provides a mutual shielding of the qubits, e.g., from stray photon exciting quasiparticles which might impair the coherence of the charge qubit.

Coupling via a superconducting cavity.—The electromagnetic modes of a superconducting cavity made of two parallel cylindrical rods are described by the phase variable $\psi(z, t)$ [10], with the Lagrangian

$$\mathcal{L} = \frac{C_r}{2L}\int_0^L dz\dot{\psi}^2 - \frac{L}{2L_r}\int_0^L dz\left(\frac{\partial\psi}{\partial z}\right)^2, \tag{4}$$

with C_r the capacitance of the cavity, L_r the inductance, and L the length. With a distance d_0 between the rods and the rod radii b_0, $L_r = \mu_0\ln(d_0/b_0)L/\pi^3$ and $C_r = 4\pi\epsilon_0 L/4\ln(d_0/b_0)$. For example, for $d_0 = 20\ \mu$m, $b_0 = 1\ \mu$m, $L = 100\ \mu$m, $C_r = 1$ fF, $L_r = 10$ pH; the frequency of the first eigenmode of the cavity $\omega_r/2\pi = 1.5$ THz. Application of a millimeter transmission line superconducting cavity in the microwave regime has been proposed for the interaction of charge qubits where the cavity mode is in resonance with the qubit [11].

The coupling scheme is shown in Fig. 1. The left end of the cavity capacitively couples with the ion as $[V_i - \dot{\psi}(0, t)]e\hat{x}/d_i$, where d_i is the distance between the ion and the cavity and V_i is the voltage on the trap electrode. The cavity couples with one of the trap electrodes by the capacitor C_i. The right end of the cavity couples with the charge qubit via a contact capacitor C_m as $C_m[\dot{\psi}(L, t) - \dot{\varphi}]^2/2$. In our scheme, the cavity length is much shorter than the wavelength of a microwave field which characterizes the energy of a charge qubit, so that the cavity can be represented by phase variables $\psi_{1,2}$ at the ends of the cavity. Each node is connected with the ground by a capacitor $C_r/2$, and the two nodes are connected by the inductor L_r, as is shown in Fig. 1. The conjugates of the phases obey the charge conservation relation $p_1 + p_2 = 0$, where $p_{1,2}$ are the total charge on each node. With the new variable $\tilde{\psi} = \psi_1 - \psi_2$ and its conjugate $\tilde{p}_\psi = (p_1 - p_2)/2$, the interaction is $H_{\rm int}^{(0)} = H_{\rm cav} + H_1$ with

$$H_{\rm cav} = \frac{\tilde{p}_\psi^2}{2(C_r/4)} + \frac{\tilde{\psi}^2}{2L_r},$$
$$H_1 = \tilde{p}_\psi\frac{e\hat{x}/d_i + C_iV_i}{C_i + C_r/2} - \tilde{p}_\psi\frac{C_m}{C_t}\frac{e\sigma_z^q + C_gV_g}{C_m + C_r/2}, \tag{5}$$

which includes the Hamiltonian of the cavity and the coupling between the cavity, the charge, and the motion, with $C_t = C_m + C_J + C_g$, and we have assumed $C_r \gg C_i, C_m, C_J, C_g$. The cavity mode is an oscillator with frequency $\omega_r = 2/\sqrt{L_r C_r}$. Effective coupling between the charge and the motion is derived with second order perturbation,

$$H_{\rm int} = \frac{e^2}{C_r}\frac{C_m}{C_t}\left(\frac{\hat{x}}{d_i} + \frac{C_i V_i}{e}\right)\left(\sigma_z^q + \frac{C_g V_g}{e}\right), \tag{6}$$

which includes the effect of the gate voltage V_g on the ion which shifts the trapping potential, and the effect of the trap voltage V_i on the charge qubit which can be avoided by designing a balance circuit (below). The cavity shortens the distance between the charge and the ion to the order of d_i with $\hbar\kappa = e^2/2C_r d_i$, where κ is increased by a factor of $4\ln(d_0/b_0)r_0/d_i$ compared with the direct coupling. With $C_g \sim C_J = 0.1$ fF and $C_i \sim C_m = 0.2$ fF, $\kappa/2\pi = 25$ GHz$/d_i$.

A fast swap gate.—A controlled phase gate, $|\epsilon_1\rangle_s|\epsilon_1\rangle_q \to (-1)^{\epsilon_1\epsilon_2}|\epsilon_1\rangle_s|\epsilon_1\rangle_q$ $(\epsilon_{1,2} = 0, 1)$, together with single-qubit rotations forms a universal set of operations required for entanglement and information exchange between the ion and the charge. The swap gate, which is the key step for interfacing the ion and the charge qubit, can be achieved by three controlled phase gates together with Hadamard gates on the qubits [12].

We construct a phase gate operating on nanoseconds, a time scale much shorter than the trap period, and not requiring the cooling of the phonon state with a eight-pulse sequence. Three evolution operators are used in this sequence. The free evolution $U_0(t) = \exp(-i\omega_\nu t\hat{a}_x^\dagger \hat{a}_x)$, where $\hat{a}_x$ ($\hat{a}_x^\dagger$) is the creation (annihilation) operator of the motion, is achieved by turning off the interactions $\kappa = \omega_R = 0$. Entanglement between the motion and the spin state is obtained by applying short laser pulses with $\omega_R\tau_s = \pi$ for n_l times ($\kappa = H_q = 0$ and n_l even): $U_l(z_l n_l) = \exp(-iz_l\delta k_l n_l \sigma_z^s \hat{x})$, where $z_k = \pm 1$ is the direction of the photon wave vector and δk_l is the momentum from one kick of the laser. The duration of the kicking τ_s is assumed to be much shorter than other time scales. Entanglement between the motion and the charge qubit is obtained by turning on the interaction κ for time τ_q ($H_q = \omega_R = 0$ and $\omega_\nu\tau_q \ll 1$): $U_2(\tau_q) = \exp(-i\kappa\tau_q\sigma_z^q\hat{x})$. By flipping the charge qubit with single-qubit operation, the sign of the evolution can be flipped as $U_2(-\tau_q) = \sigma_x^q U_2(\tau_q)\sigma_x^q$.

The gate sequence is

$$\begin{aligned} U(T) = {} & U_l(n_l^2)U_q(\tau_q^2)U_0(t_2)U_l(-n_l^1 - n_l^2)U_q(-\tau_q^1 - \tau_q^2) \\ & \times U_0(t_1)U_l(n_l^1)U_q(\tau_q^1), \end{aligned} \tag{7}$$

where the parameters fulfill $n_l^1 t_1 = n_l^2 t_2$ and $\tau_q^1 t_1 = \tau_q^2 t_2$. When $\omega_\nu t_{1,2} \ll 1$, by making the approximation $\exp(-i\omega_\nu t) \to 1 - i\omega_\nu t$,

$$U(T) = e^{i\phi'}U_0(t_1 + t_2)e^{-i\alpha\sigma_z^q\sigma_z^s}, \tag{8}$$

where ϕ' is a global phase, $k_{\rm eff}^i = \delta k_l n_l^i \sigma_z^s + \kappa\tau_q^i\sigma_z^q$ with $i = 1, 2$, and $\alpha = \hbar\kappa\delta k_l \tau_q^1 n_l^1 t_1/mt_2(t_1 + t_2)$. The motional part factors out from the evolution of the qubits. Hence, the gate does not depend on the initial state of the phonon mode. The fidelity is $1 - O(\omega_\nu^2 t^2)$ and can be improved by exploiting a low trapping frequency. Note for the free particle ($\omega_\nu = 0$), Eq. (8) is exact. For the phase gate $\alpha = \pi/4$. This gives the total gate time

$$T = \frac{\pi m}{4\hbar\kappa\delta k_l}\left(\frac{1}{n_l^1 t_1} + \frac{1}{n_l^2 t_2}\right) + t_1 + t_2, \tag{9}$$

which shows that the limits of the phase gate are essentially set by the available Rabi frequency of the laser and the coupling κ. We choose $t_1 = t_2 = 5$ nsec. With $\delta k_l = 10^8$ m^{-1}, $n_l^{1,2} = 10$, for ^{9}Be$^+$, the gate time is $T = 14$ nsec; and for ^{43}Ca$^+$, the gate time is $T = 26$ nsec, much shorter than the decoherence time of the qubits.

Decoherence of the combined system.—In the interacting system of the ion, the charge qubit, and the cavity, decoherence of any component affects the dynamics of the others. Decoherence of the ion trap qubit [13] and the superconducting charge qubit [4] have been well studied and a coherence time of microseconds has been measured. Here we concentrate on decoherence introduced by the cavity, which is the new element in the scheme, in particular, the effect of excitation of quasiparticles in the superconducting cavity generated by stray laser photons.

The dissipation of the cavity is described by a resistor R_r in series to the inductance L_r. Following the two fluid model [14,15], the resistance is $R_r = \sigma_1 L/\lambda\sigma_2^2 b_0$, where $\sigma_1 = n_n e^2\tau_n/m$ with n_n the density of quasiparticles and τ_n the mean free time, $\sigma_2 = n_s e^2/m\omega$ with n_s the density of superconducting electrons, and λ is the penetration depth (typically of order of microns). Without radiation, $n_n \sim n_0\exp(-2\Delta/k_B T)$ (n_0 is total electron density) yields negligible resistance, when temperature is well below the superconducting gap Δ, and negligible dissipation. However, stray photons from the ion trap excite quasiparticles and lead to a resistance of $R_r = R_n(n_{\rm ex}/n_0)$, where $n_{\rm ex}$ denotes excited quasiparticles and $R_n \sim 10^4$ Ω with given parameters is the normal state resistance. Note the cavity is cooled to a temperature below the gap. For bulk aluminum, $\Delta = 4.2$ K; temperature lower than Kelvin is required. At high temperature, e.g., room temperature for a normal ion trap, the electrodes enter the normal metallic state and dissipation is characterized by normal state resistance R_n.

The noise spectral density of the cavity loss can be derived with an imaginary time path integral approach [16] when modeled as a bosonic bath:

$$J(\omega) = \left(\frac{C_m}{2C_t}\right)^2 \omega Z_{\rm eff}(\omega)\coth\left(\frac{\hbar\omega}{2k_B T}\right), \tag{10}$$

where the effective impedance $Z_{\rm eff}$ is a capacitor $(C_r + C_m)/4$ in parallel to the series of the inductor L_r and the resistor R_r. With $\omega \ll 1/\sqrt{L_r C_r}$, $Z_{\rm eff} \approx R_r$. With the fluctuation-dissipation theorem (FDT), the decoherence rate of the charge and the motion can be derived from the spectral density

$$\gamma_r^q \approx \frac{R_r}{R_k}\frac{2k_BT}{\hbar}\left(\frac{C_m}{2C_t}\right)^2, \qquad \gamma_r^x \approx \frac{R_r}{R_k}\frac{2k_BT}{\hbar}\left(\frac{x_r}{4d_i}\right)^2, \quad (11)$$

respectively, where $R_k = \hbar/(2e)^2$ is the quantum resistance and x_r is the spatial displacement of the dipole. Considering the laser power of mW, and assuming the absorbed power to be nW for a duration of 100 nsec, $R_r = R_n/10^5$. With temperature $T = 100$ mK, $\gamma_r^q = 50$ msec^{-1} and $\gamma_r^x = 5$ sec^{-1}. This shows that the dominant decoherence is not by the cavity loss.

Discussion.—Combining two drastically different systems naturally introduces technical questions of compatibility, such as the coexistence of an ion trap with a cavity and connected charge qubit. Ions can be trapped either with a Paul or a Penning trap, i.e., employing strong electric or magnetic fields, while a mesoscopic charge qubit cannot survive a magnetic field exceeding ~0.1 Tesla and a voltage exceeding ~1 mV. In the case of a Paul trap typically radio frequency fields up to 250 MHz are applied which, according to Eq. (6), couples to the charge qubit via the capacitor C_i. For example, trapping a single $^{43}Ca^+$ ($^9Be^+$) ion in a trap of the size of ~20 μm ring diameter (or cap distance) requires $V_{\rm trap}$ about 30–50 V at 100–250 MHz to achieve a trap depth of about 1–1.5 eV with corresponding trap frequencies of 18–20 MHz. Thus capacitive coupling of the trap's drive frequency to the end caps must be carefully compensated for by using tailored electronic filter circuits. This is only schematically indicated in Fig. 1; in all experimental setups higher order filtering is routinely used. Thus, the voltage couples to the charge qubit is now $C_iV_i + C_{ib}V_{ib} \rightarrow 0$ where only a small residue voltage due to imperfect circuitry passes to the charge qubit. With a residue of 0.1 V which is far off resonance, the dynamics of the qubit is not affected significantly. We note that the balance circuit requires refined electronic filtering and feedback control circuitry. In the case of a Penning trap, by using a superconducting thin film that sustains high magnetic field or by using a cavity geometry that separates the qubit from the trap, the qubit can coexist with the trap.

Coupling of two ions via a cavity.—Instead of coupling an ion to a charge qubit via a cavity, we can also couple two ions, albeit at the expense of a reduced coupling strength. This provides an alternative to the standard scenarios of scalable quantum computing with trapped ions, which are based on moving ions [2]. Note that ion coupling via trap electrodes has been proposed for Penning traps in [6]. With the geometry according to Fig. 1, the ions couple to the ends of the cavity. The ion-ion Hamiltonian can be derived as

$$H_{i-i} = H_s^1 + H_s^2 + \frac{e^2}{2(C_r + 2C_i)}\frac{\hat{x}_1\hat{x}_2}{d_i^2}, \qquad (12)$$

where $H_s^{1,2}$ are the Hamiltonians for the two ions defined in Eq. (1), and the trap voltage can be included by replacing $e\hat{x}_{1,2}/d_i \rightarrow e\hat{x}_{1,2}/d_i + C_iV_i$. Compared with the direct (free space) coupling between two dipoles with a distance L, the interaction is enhanced by a factor of $4\ln(d_0/b_0)(L/d_i)^2$. Besides, the coupling has the advantage of being switchable: by inserting a switch, e.g., a tunable Josephson junction, the interaction can be turned on and off rapidly. The coupling is, in principle, scalable by fabricating multiple connected cavities.

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Note added.—After completion of this work we became aware of the Letter by Sørensen *et al.* [17] discussing coupling Rydberg atoms by transmission line.

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