QUANTUM MECHANICS

Dynamics of entanglement

Quantum entanglement is a vital resource in quantum information science. A theoretical framework now provides a better understanding of how these non-classical correlations decay in a real environment.

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Quantum information theory teaches us how to solve certain computation or communication problems efficiently by basing them on quantum principles, in particular on the ability to create and manipulate quantum entanglement between different parts of a system[1]. But experimentalists who work towards bringing such ideas to fruition by constructing suitable quantum devices will invariably encounter another kind of entanglement, originating from interactions of the quantum system under consideration with uncontrolled degrees of freedom, usually termed 'environment'. These undesired couplings will give rise to a loss of the entanglement present between different subsystems of the quantum device. Therefore, it is of great practical importance to find tools that enable the characterization of the decohering interactions between a quantum system and its environment. On page 99 of this issue, Thomas Konrad and colleagues[2] report such a tool for the basic, but relevant case of two entangled quantum bits (qubits), one of which is interacting with an environment.

The interaction between a qubit and its environment can be described as a 'quantum channel' (Fig. 1a,b). This is the general notion for any device operating under the rules of quantum mechanics that takes a quantum system as input and maps it to another quantum system at its output. For example, in quantum communication, a quantum channel could be given by an optical fibre delivering the quantum state of a photon from one place to another. But a quantum channel does not need to link two points in space. It can also describe the dynamical action of a quantum gate in a quantum computer, or that of a decohering environment on a qubit.

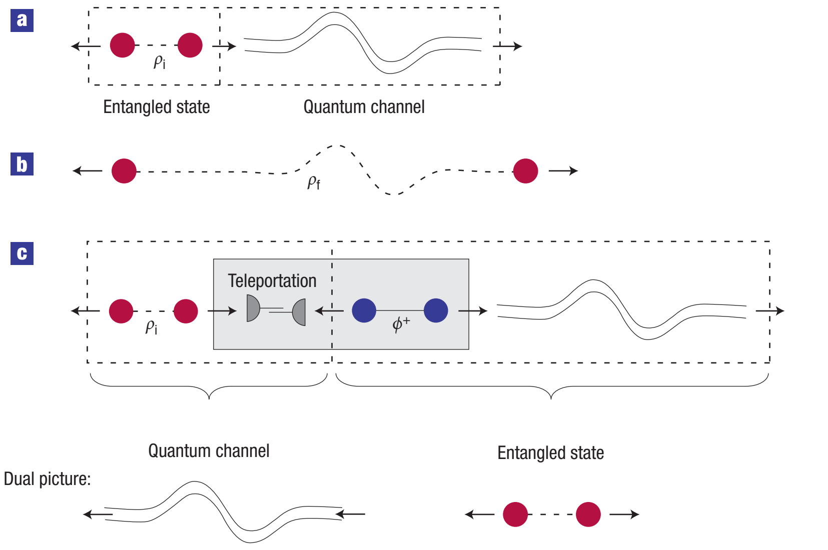

Figure 1 Sending entanglement through a quantum channel. **a**, A source (denoted by two outgoing arrows) creates a pair of qubits in an entangled state ρ_i. Subsequently, one of the qubits is sent through a quantum channel (indicated by an ingoing and an outgoing arrow). **b**, The quantum channel transforms the initial state into a state ρ_f that in general will be less entangled than the initial state. **c**, Quantum teleportation[7] consists of a joint measurement (represented by two detectors, shown in grey) on the input state and a maximally entangled state ϕ^+. This operation can be considered a quantum channel that corresponds to an identity operation. Inserting this channel into the configuration shown in **a** allows the set-up to be reinterpreted in a way such that the roles of the quantum state and the quantum channel are interchanged (the source of entangled states now being on the right-hand side, consisting of maximally entangled states propagated through the original channel).

Entanglement is the key ingredient of many quantum information protocols. The question to what extent a quantum channel preserves the entanglement between two quantum systems when one of them is sent through the channel is therefore of particular interest. Answering this question requires, first of all, having a measure capable of quantifying the amount of entanglement contained in a composite quantum state[3]. For the case of two qubits prepared in an entangled state ρ_i, the so-called concurrence $C(\rho_i)$ provides a measure of the entanglement by a closed algebraic expression specifying the nonlinear dependence of C on ρ_i (ref. 4). The entanglement loss caused by the quantum channel can be inferred in a two-step process. First, the evolution of the quantum state under the action of the quantum channel needs to be calculated. In a second step, the concurrences of the initial and final state, $C(\rho_i)$ and $C(\rho_f)$, are evaluated to obtain the amount of entanglement reduction, $C(\rho_f)/C(\rho_i)$. In an experiment, this procedure could be implemented by determining the initial and final quantum state by a technique known as quantum state tomography[5].

Although this 'naive' approach provides a valid answer, it has the drawback that it needs to be repeated every time a different input state is chosen. Konrad *et al.*[2] look at the problem from a fresh perspective (Fig. 1c). Instead of calculating the time evolution for each input state separately, they consider the dynamics of one particular state — the so-called maximally entangled state — and derive from this analysis general statements valid for arbitrary input states. As an important tool, they used a one-to-one mapping between quantum channels and quantum states that has been known for more than thirty years[6], and has many applications in quantum information theory. Konrad and colleagues[2] use this isomorphism to exchange the role of the quantum state and the quantum channel and demonstrate that in this 'dual picture', the original problem can be solved more efficiently. For pure input states, they arrive at a factorization law stating that the entanglement of the final state, ρ_f, can be expressed as the product of the initial entanglement, $C(\rho_i)$, multiplied by the entanglement reduction induced by the passage through the quantum channel; the latter turns out to be universal for all quantum states entering the channel. These findings can be generalized to mixed input states and to concatenation of quantum channels, at the price, however, of turning the factorization law into an inequality. Most importantly, the new technique provides a direct way of inferring the dynamical evolution of entanglement under the action of a decohering environment.

The work by Konrad *et al.*[2] elegantly deals with the case of two entangled qubits. But further ideas will be necessary to extend the results to general composite quantum systems as the derivation relies heavily on the availability of an entanglement measure; for two qubits, such a measure is easy to calculate, but no equivalent is available for higher-dimensional systems. This notwithstanding, the results of Konrad and colleagues add an important piece to our understanding of entanglement and its dynamical aspects.

References

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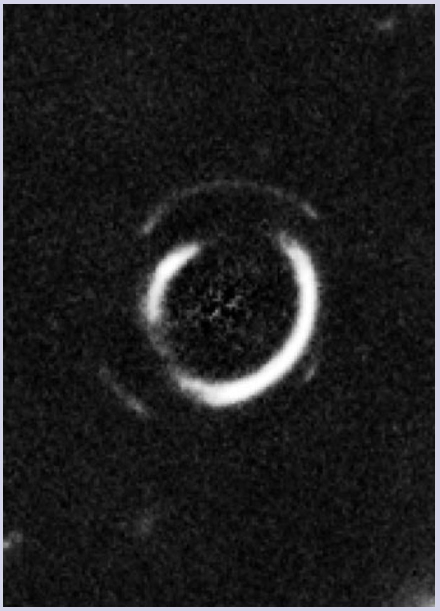
ASTROPHYSICS

Rings around the lenses

During a routine Hubble Space Telescope scan for an Einstein ring — a galaxy with a bright ring around it — astronomers noticed a double ring, the first of its kind. The team led by Raphael Gavazzi and Tommaso Treu report their discovery online (http://arxiv.org/abs/0801.1555; 2008).

A single Einstein ring is a consequence of gravitational lensing, whereby a massive galaxy (the lens) bends the light from a more distant galaxy (the object) along the same line of sight, or optical axis. Rather than focusing the light, as lenses do, the galaxy in the foreground has a focal line that creates mirages of the object, so we see the deflected light as a ring around the lens instead of a spot. (When the alignment is not perfect, arcs will appear instead of a full ring.) Acting as a magnifier, the lens amplifies the brightness of the object.

Albert Einstein himself made the calculation for lensing by a single star, which didn't yield an easily observable ring, but Fritz Zwicky was the first to propose that galaxies can also act as gravitational lenses. In fact, any massive object can bend space–time.



It's a way to observe the unobservable: we can't see a black hole or dark matter, but we can see the effect they have on other objects.

In turn, the lensing provides a way for astronomers to determine the mass distribution of the lens galaxy. For a double ring, which is produced by light from not one but two distant galaxies beyond the one we can see, the situation is even better. The galaxy in the middle acts as an additional lens for the most distant one (to create the outer ring). Of course, the compound lens is more difficult to model, but it provides a unique method of measuring the total mass of small distant galaxies.

At 3 billion, 6 billion and approximately 11 billion light-years away, the galaxies in the double Einstein ring are cosmologically distant. A natural question then is whether we can obtain constraints on cosmological parameters. According to Gavazzi *et al.,* the uncertainties are currently too large for any meaningful interpretations. However, they calculate that fifty such double-source lens systems would lead to measurements of the matter density of the Universe and the equation of state of dark energy with unprecedented accuracy (10%). With several planned space missions expected to reveal tens of thousands of single Einstein rings and tens of double Einstein rings, that goal may not be so far away.

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